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Calculation of Resonant Frequencies and Electromagnetic Fields in Resonators of Linear Accelerators for Commercial Application, Medicine and Environmental Protection

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ABSTRACT

Numeric simulation in studies of electrodynamic processes in charged particle accelerators becomes more and more popular. The tendency towards increase in current of charged particles and total unit capacity has lead to qualitatively new requirements to numeric simulation of electrodynamic processes. One of the most significant requirements is multi-mode type of models, that is, possibility to investigate into interaction between charged particle beams and a set of electromagnetic oscillations, which are excited in operation spaces. In this work numeric model is based on electrodynamic simulation of slowing systems in the form of a chain of coupled cavity resonators, and electromagnetic fields of the system are described by expanding on major basis which is generated in complex shape resonator using numeric method.
Keywords: numeric simulation, slowing system, linear accelerator, beam emittance, higher order modes, multi-mode models, cavity resonator, electromagnetic field, equipotential.

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INTRODUCTION

Current development of charged particle accelerators is characterized with increased requirements to parameters pf accelerated beam. This is stipulated by expansion of the scope of physical problems studied by means of charged particle beam. It this regard it would be reasonable to apply the notion of beams with precious parameters. Such characteristics as narrow energy spectrum, low values of longitudinal and transversal emittance, high values of short- and long-term stability of beam energy and current should be implemented at high energy and significant intensity of accelerated particles.

In addition to conventional application of charged particle accelerators as a tool of experimental physics in recent years the fields of their application in national economy are expanded (Ryabukhin et al., 1980). Certain requirements to such application accelerators are specified: ease of maintenance, economic efficiency, reliability.

In this regard numerical experiment gains high practical importance, which in some cases is significantly less expensive than natural experiment and does not require for high expenses. Moreover, as practically shown, numerical experiment makes it possible to reveal previously unknown processes and phenomena, to serve as a tool of sophisticated and comprehensive investigations into simulated subject (Roshal', 1979).

Numerous methods are developed and applied in order to obtain accelerated beams with required properties (Gusarova et al., 2009). Among others they involve parametric stabilization of properties of accelerator systems, application of systems of beam generation at various accelerating stages from injection to its extraction onto target. Accelerating system plays significant role in obtaining of required beam properties. In accelerating sections clusters are generated and accelerated, their sizes and shape are varied. Investigation into electrodynamic parameters of accelerating sections enables development of methods of beam quality improvement and obtaining of its required characteristics.

Obtaining of required beam characteristics to a considerable extent is determined by the influence of higher order modes, since during passing of charged particle beam in accelerating structure wide spectrum of waves is generated with various resonance frequencies. This effect impacts negatively on accelerator operation, since in this case a portion of beam energy is consumed by electromagnetic field, however, more significant is the interaction between induced waves with the beam (Bolgov et al., 2013).

The induced fields of higher order modes have transversal electric and magnetic components on the axis and, thus, deviate subsequent clusters. This can cause significant increase in transversal and longitudinal beam emittance and finally lead to expansion of energy spectrum of particles and even to loss of particles on the walls, and in the case of increase in beam current to its complete loss.

Damping of higher order modes plays an important role in achievement and retention of low emittance and low modulation of beam energy in accelerators, especially in accelerators on the basis of superconducting technology.

Nowadays, as a consequence of on-going projects aimed at development of high-current electron and ion accelerators the application of methods of computer aided electrodynamics becomes more widely applied at the stages of estimations and preliminary analysis of the processes in accelerating structures. Increase in beam current and total capacity of accelerators lead to qualitatively new requirements specified for simulation of the processes.

One of such requirements is multi-mode type of models, that is, possibility to investigate into interaction between charged particle beams and a set of electromagnetic oscillations, which are excited during beam passing across the considered subjects or during transmission of electromagnetic энергии from generator to beam.

The most complete generation of such model can be achieved by means of electrodynamic approach, when the problem is solved numerically either directly with the Maxwell equations (Roshal', 1979), or with equations acquired from the Maxwell equations by approximations and restrictions (Slater, 1948).

One of the problems of computer aided electrodynamics upon generation of multi-mode model is determination of electrodynamic properties of oscillations, which can be excited in the spaces where filed interacts with flows of charged particles.

This work proposes a method of calculations of electromagnetic fields and resonance frequencies of higher order modes in complex-shape resonators and other important parameters of accelerating structure.

EXPERMIMENTAL

Calculation of resonance frequencies and electromagnetic fields of higher order modes oscillations higher order modes.

Let us consider a resonator, which represents certain space restricted with planes of symmetry, if any, and ideally conducting surface of complex shape, such that it is not a coordinate plane in known coordinate systems, which facilitate solution of wave equation by partition method.

It is required to determine electrodynamic properties, such as eigen frequency, Q-factor, distribution of electric and magnetic fields of such oscillations, which can be excited in this resonator.

In order to solve the set problem let us generate Для решения поставленной задачи построим в исследуемом объёме curvilinear orthogonal coordinate system in the considered space, such that the restricted surface becomes the coordinate plane. For vacuum space without extraneous currents the Maxwell equations can be written as follows:

$$rot\boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t}, rot\boldsymbol{H} = \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}, div\boldsymbol{E} = 0, div\boldsymbol{H} = 0 \qquad (1)$$

Assuming that the fields depend harmonically on time, which is not a restriction upon identification of this problem, we obtain as follows :

$$rot\boldsymbol{E} = j\omega\mu_0\boldsymbol{H}, rot\boldsymbol{H} = j\omega\varepsilon_0\boldsymbol{E}. \qquad (2)$$

Let us consider the obtained equations in curvilinear orthogonal ой orthogonal ой coordinate system (u^1, u^2, u^3). In covariant components:

$$\boldsymbol{E} = \boldsymbol{a^1}e_1 + \boldsymbol{a^2}e_2 + \boldsymbol{a^3}e_3, \boldsymbol{H} = \boldsymbol{a^1}h_1 + \boldsymbol{a^2}h_2 + \boldsymbol{a^3}h_3, \qquad (3)$$

where $\boldsymbol{a^1}, \boldsymbol{a^2}, \boldsymbol{a^3}$ are the relative unit vectors.

Introducing covariant representations of field vectors (3) into Eq. (2) we obtain as follows:

$$\begin{cases} 1/\sqrt{g}\left[\boldsymbol{a^1}\left(\frac{\partial e_3}{\partial u^2} - \frac{\partial e_2}{\partial u^3}\right) + \boldsymbol{a^2}\left(\frac{\partial e_1}{\partial u^3} - \frac{\partial e_3}{\partial u^1}\right) + \boldsymbol{a^3}\left(\frac{\partial e_2}{\partial u^1} - \frac{\partial e_1}{\partial u^2}\right)\right] = -j\omega\mu_0(\boldsymbol{a^1}h_1 + \boldsymbol{a^2}h_2 + \boldsymbol{a^3}h_3), \\ 1/\sqrt{g}\left[\boldsymbol{a^1}\left(\frac{\partial h_3}{\partial u^2} - \frac{\partial h_2}{\partial u^3}\right) + \boldsymbol{a^2}\left(\frac{\partial h_1}{\partial u^3} - \frac{\partial h_3}{\partial u^1}\right) + \boldsymbol{a^3}\left(\frac{\partial h_2}{\partial u^1} - \frac{\partial h_1}{\partial u^2}\right)\right] = j\omega\varepsilon_0(\boldsymbol{a^1}e_1 + \boldsymbol{a^2}e_2 + \boldsymbol{a^3}e_3), \end{cases} \qquad (4)$$

where $\boldsymbol{a^1}, \boldsymbol{a^2}, \boldsymbol{a^3}$ are the respective coordinate unit vectors; $g = g_{11}g_{22}\ g_{33}$ is the basic determinant of orthogonal coordinate system.

Multiplying both members of Eq. (4) by $\boldsymbol{a^1}$ and applying the ratios of curvilinear orthogonal coordinate we obtain for the first equation:

$$\begin{cases} \dfrac{\partial e_3}{\partial u^2} - \dfrac{\partial e_2}{\partial u^3} = j\omega\varepsilon_0 g_1 h_1, \\ \dfrac{\partial e_1}{\partial u^3} - \dfrac{\partial e_3}{\partial u^1} = j\omega\varepsilon_0 g_2 h_2, \\ \dfrac{\partial e_2}{\partial u^1} - \dfrac{\partial e_1}{\partial u^2} = j\omega\varepsilon_0 g_3 h_3, \end{cases} \quad (5)$$

where: $g = \sqrt{\frac{g_{22}g_{33}}{g_{11}}}, g = \sqrt{\frac{g_{11}g_{33}}{g_{22}}}, g = \sqrt{\frac{g_{22}g_{11}}{g_{33}}}$.

Similarly, for the second equation:

$$\begin{cases} \dfrac{\partial h_3}{\partial u^2} - \dfrac{\partial h_2}{\partial u^3} = -j\omega\mu_0 g_1 e_1, \\ \dfrac{\partial h_1}{\partial u^3} - \dfrac{\partial h_3}{\partial u^1} = -j\omega\mu_0 g_2 e_2, \\ \dfrac{\partial h_2}{\partial u^1} - \dfrac{\partial h_1}{\partial u^2} = -j\omega\mu_0 g_3 e_3, \end{cases} \quad (6)$$

Using Eqs. (5) and (6), it is possible to write a set of three differential equations of the second order with regard to the components of electric and magnetic fields. As an example, let us consider generation of such set of equations for spaces with axial symmetry. It can be easily seen that axial symmetry of the space lead to harmonic dependence of field components on azimuth. If the curvilinear orthogonal coordinate system generated in the considered space also has axial symmetry and the coordinate u^2 is similar to azimuth coordinate in cylindrical coordinate system, then the following equations are valid:

$$\begin{cases} h_1 = h_1(u^1, u^3) \cdot f(mu^2), e_1 = e_1(u^1, u^3) \cdot f'(mu^2), \\ h_2 = h_2(u^1, u^3) \cdot f'(mu^2), e_2 = e_2(u^1, u^3) \cdot f(mu^2), \\ h_3 = h_3(u^1, u^3) \cdot f(mu^2), e_3 = e_3(u^1, u^3) \cdot f'(mu^2). \end{cases} \quad (7)$$

In Eq. (7): $m = 1, 2, 3$ is the number of field variations in azimuth; $f(mu^2) = \sin(mu^2)$ or $f(mu^2) = \cos(mu^2)$.

Applying simple transformations, using Eqs. (5), (6) and with consideration for Eq. (7) it is possible to obtain equations with regard to three components of magnetic field, where k is the wave number:

$$\begin{cases} \dfrac{m^2}{g_3}h_1 - \dfrac{1}{g_2}\dfrac{\partial^2 h_1}{(\partial u^3)^2} - \dfrac{\partial h_1}{\partial u^3}\dfrac{\partial}{\partial u^3}\left(\dfrac{1}{g_2}\right) - k^2 g_1 h_1 - \dfrac{m}{g_3}\dfrac{\partial h_2}{\partial u^1} + \dfrac{1}{g_2}\dfrac{\partial^2 h_3}{\partial u^2 \partial u^3} + \dfrac{\partial h_3}{\partial u^1}\dfrac{\partial}{\partial u^3}\left(\dfrac{1}{g_2}\right) = 0, \\ \dfrac{m}{g_3}\dfrac{\partial h_1}{\partial u^1} + m h_1 \dfrac{\partial}{\partial u^1}\left(\dfrac{1}{g_3}\right) - \dfrac{1}{g_1}\dfrac{\partial^2 h_2}{(\partial u^3)^2} - \dfrac{\partial h_2}{\partial u^3}\left(\dfrac{1}{g_1}\right) - \dfrac{1}{g_3}\dfrac{\partial^2 h_2}{(\partial u^1)^2} - \dfrac{\partial h_2}{\partial u^1}\dfrac{\partial}{\partial u^1}\left(\dfrac{1}{g_3}\right) - k^2 g_2 h_2 + \\ + \dfrac{m}{g_1}\dfrac{\partial h_3}{\partial u^3} + m h_3 \dfrac{\partial}{\partial u^3}\left(\dfrac{1}{g_1}\right) = 0, \\ \dfrac{1}{g_2}\dfrac{\partial^2 h_1}{\partial u^1 \partial u^3} + \dfrac{\partial h_1}{\partial u^3}\dfrac{\partial}{\partial u^1}\left(\dfrac{1}{g_2}\right) - \dfrac{m}{g_1}\dfrac{\partial h_2}{\partial u^3} - \dfrac{1}{g_2}\dfrac{\partial^2 h_3}{(\partial u^1)^2} - \dfrac{\partial h_3}{\partial u^1}\dfrac{\partial}{\partial u^1}\left(\dfrac{1}{g_2}\right) + \dfrac{m}{g_1}h_3 - k^2 g_3 h_3 = 0. \end{cases} \quad (8)$$

Substituting differential operators for difference analogs according to the five-point approximation scheme (it is optimum for this problem), it is possible to obtain a set of linear algebraic equations with regard to unknown components of magnetic field in the nodes of curvilinear orthogonal grid.

It should be mentioned that setting of boundary conditions of any type is not very complicated, since the resonator boundary is coordinate and the vector components are either normal or tangential with regard to restricting surface. The matrix of such obtained set of linear algebraic equations is in general case 15-diagonal, though upon transition from differential operator to difference analog, as can be easily demonstrated, the maximum number of unknown variables in each equation of the set (8) upon five-point approximation decreases to eleven.

The obtained set of linear algebraic equations is homogeneous, hence, non-trivial solution will exist only at such wave numbers when the determinant is zero. Therefore, the problem of determination of resonance frequencies of oscillations can be reduced to determination of determinant zeroes. It should be mentioned that the determinant dependence on frequency for this set of algebraic equations is alternating differentiable function with continuous derivatives up to the N^{th} order (N is the number of grid nodes), which facilitates development of algorithm of zero det4ermination without operator interference.

In order to calculate determinant of the obtained set of equations it is possible to apply the method of Gaussian elimination, with its adjustment for certain form of the matrix of the set coefficients, which leads to significant reduction of calculations.

Interrelation between geometrical sizes of slowing structure of accelerator with electrodynamic characteristics.

Let us obtain expression interrelating geometrical sizes of biperiodic slowing structure (BSS) and values of accelerated currents with its electrodynamic characteristics. With this aim let us consider an arbitrary resonator representing certain space V, restricted with closed surface S. This surface is the metallic shell of resonator with cut holes for interaction with other resonators or inlet waveguide. We consider the determination of electromagnetic fields excited in resonator at preset frequency ω.

Since the internal resonator surface can be sufficiently complex, then an accurate solution of the Maxwell equations for the considered space is impossible. In this regard let us represent the required solution in the form of sum of normal oscillations, which are orthogonal.

It was demonstrated theoretically and experimentally (Knapp et al., 1968), that BSS sensitivity to current loading and other types of detuning is the lower the higher is the distance of working type of oscillations to adjacent types of oscillations, and that dispersion curve in general case is split into two branches subdivided by stop band (Wasow and Forsythe, 1963). Existence of stop band leads to distribution heterogeneity of electric field in accelerating cells and decrease in effective shunt resistance due occurrence of electric field in connection cells. Thus, for determination of BSS sensitivity to perturbation factors it is required to know dispersion dependence.

It is known (Slater, 1948) that any solution of the Maxwell equations for resonator contains in general case solenoidal (vortex) and potential (gradient) components. Let us be restricted only by solenoidal component which is usually considered as "radiation field".

Let us select a certain system of solenoidal functions $(\boldsymbol{E}_m, \boldsymbol{H}_m)$, (where *m* = 1,2…), according to which the required fields $\boldsymbol{E}$ and $\boldsymbol{H}$ are expanded in resonator. The selected set of functions satisfies wave equation with homogeneous boundary conditions. Thus, the set of solenoidal functions $(\boldsymbol{E}_m, \boldsymbol{H}_m)$ is a solution of the Maxwell equations in resonator, the surface of which is ideal conductor. Hence, it is possible to state that under these assumptions the selected set generates complete orthonormalized set of functions (Mashkovtsev *et al.*, 1966)

$$\int_V \boldsymbol{E}_k \boldsymbol{E}_m \, dV = \int_V \boldsymbol{H}_k \boldsymbol{H}_m \, dV = \delta_{km} = \begin{cases} 0, k \neq m \\ 1, k = m \end{cases}. \quad (9)$$

In this case it is possible to obtain the following set of equations for required electric field of the considered resonator:

$$\frac{d^2}{dt^2}\int_V \boldsymbol{E}\boldsymbol{E}_m \, dV + \omega_m^2 \int_V \boldsymbol{E}\boldsymbol{E}_m \, dV =$$
$$-\frac{\omega_m}{\sqrt{\varepsilon\mu}}\int_S [\boldsymbol{n}\boldsymbol{E}]\, \boldsymbol{H}_m dS - \frac{1}{\varepsilon}\frac{d}{dt}\int_V \boldsymbol{J}\, \boldsymbol{E}_m dV - \frac{\omega_m}{\sqrt{\varepsilon\mu}}\int_{S_{hc}} [\boldsymbol{n}\boldsymbol{E}]\, \boldsymbol{H}_m dS\,. \quad (10)$$

Here $\boldsymbol{E}_m$, $\boldsymbol{H}_m$, ω_m are the eigen functions and eigen frequencies of the resonator; $\boldsymbol{J}$ is the vector of current density in arbitrary point of resonator space; t is the time; S_{hc} is the cumulative surface area of connection holes of the resonator; ε, μ are the absolute dielectric and magnetic permeability.

Similar equation can be obtained for magnetic field. The obtained Eq. (10) can be generalized for the case of a chain of coupled resonators generating BSS, herewith, the respective variables will be denoted with the index *n* (resonator number):

$$\frac{d^2}{dt^2}\int_{V_n}\boldsymbol{E}_n\boldsymbol{E}_{nm}dV+\omega_{nm}^2\int_V\boldsymbol{E}_n\boldsymbol{E}_{nm}dV$$
$$=-\frac{\omega_{nm}}{\sqrt{\varepsilon\mu}}\int_S[\boldsymbol{n}\boldsymbol{E}_n]\,\boldsymbol{H}_{nm}dS-\frac{1}{\varepsilon}\frac{d}{dt}\int_V\boldsymbol{J}_n\,\boldsymbol{E}_{nm}\,dV-\frac{\omega_{nm}}{\sqrt{\varepsilon\mu}}\int_{S_{hc}}[\boldsymbol{n}\boldsymbol{E}_n]\,\boldsymbol{H}_{nm}dS\,.\,(11)$$

The order of the set is determined as the product of number of resonators in the chain by the number of the taken functions of the set $(\boldsymbol{E}_m, \boldsymbol{H}_m)$ in expansion of the fields $\boldsymbol{E}$ and $\boldsymbol{H}$.

The Eq. (11) is the required set for the chain of coupled resonators. Solution of this set makes it possible to determine general electrodynamic characteristics of slowing structure generated by the chain of coupled resonators. They are as follows: dispersion, distribution of fields across the structure, as well as external parameters (inlet resistance, reflection coefficient and others).

Let us assume that the slowing structure is not connected with inlet waveguide and does not contain accelerated particles. In this case the second term in the right side of Eq. (11), reflecting interaction of accelerated particle beam with the resonator field, is zero, and S_{hc} is only the surface of resonator connection holes between themselves. Then, the required fields in *n*-resonator of slowing structure can be presented as follows:

$$\boldsymbol{E}_n(r,t)=Im[\boldsymbol{E}_n(\boldsymbol{r})e^{-i\omega t}]=\sum_{m=1}^{\infty}Im\,[V_{nm}\boldsymbol{E}_{nm}(\boldsymbol{r})e^{-i\omega t}],\qquad(12)$$
$$\boldsymbol{H}_n(r,t)=Im[\boldsymbol{H}_n(\boldsymbol{r})e^{-i\omega t}]=\sum_{m=1}^{\infty}Im\,[I_{nm}\boldsymbol{H}_{nm}(\boldsymbol{r})e^{-i\omega t}],\qquad(13)$$

where: V_{nm}, I_{nm} are the unknown amplitude expansion coefficients of the fields $\boldsymbol{E}$ and $\boldsymbol{H}$, $\boldsymbol{r}$ is the radius vector of the considered point in *n*-resonator.

Then, with consideration for Eqs. (9), (12) and (13) the left side of Eq. (11) can be transformed as follows:

$$\frac{d^2}{dt^2}\int_{V_n}\boldsymbol{E}_n\boldsymbol{E}_{nm}\,dV+\omega_{nm}^2\int_V\boldsymbol{E}_n\boldsymbol{E}_{nm}\,dV=Im[(\omega_{nm}^2-\omega^2)(V_{nm}e^{-i\omega t}].\qquad(14)$$

The first term in the right side of Eq. (11) reflects loss in the resonator walls and can be expressed by means of the value of its own Q-factor Q_{nm}:

$$\frac{\omega_{nm}}{\sqrt{\varepsilon\mu}}\int_{S_n}[\boldsymbol{E}_{\boldsymbol{n}}(\boldsymbol{r})\boldsymbol{H}_{nm}]\,\boldsymbol{n}dS=\omega^2(i-1)\frac{1}{Q_{nm}}V_{nm}\,.\qquad(15)$$

The integrals over the surface of connection holes S_{hc} in Eq. (11) reflect excitation of *n*-resonator by electromagnetic fields in connection holes with adjacent resonators or with inlet waveguides трактом, if it is connected to *n*-cell. In the considered case the electric connection between resonators can be neglected and the surfaces of connection holes can be presented in integral form as follows:

$$\int_{S_{hc}}[\boldsymbol{n}\boldsymbol{E}_n]\,\boldsymbol{H}_{nm}dS=\int_{S_n}\left[\boldsymbol{E}_{tgn}\boldsymbol{H}_{nm}\right]\boldsymbol{n}dS\,.\qquad(16)$$

Substituting the obtained Eqs. (14), (15) and (16) into Eq. (11) we obtain a set of equations with rgard to unknown coefficients V_{nm}

$$(\omega_{nm}^2 - \omega^2)V_{nm} = \omega^2(1-i)\frac{1}{Q_{nm}}V_{nm} - \frac{\omega_{nm}}{\sqrt{\varepsilon\mu}}\int_{S_n}\left[\boldsymbol{E}_{tgn}\boldsymbol{H}_{nm}\right]\boldsymbol{n}dS\,. \quad (17)$$

The values to be instantiated and defined in Eq. (17) include tangential component of electric field on connection hole $\boldsymbol{E}_{tgn}$**.** The considered BSS contain axial symmetric resonators, in each of them narrow connection gaps are cut in side walls in direction of azimuth coordinate.

Since the gap width is by far lower than the wave length, then it makes possible to represent the gap by transmitting line without loss, short-circuited at the ends. Along this line a wave of T type is propagated, herewith, at the left and right planes of the gap only tangential component of electric field exists. Such assumption is in direct logical compliance with neglecting of potential portion of the field.

The distributed capacitance of the gap is determined by solution of electrostatic problem in the plane perpendicular to the direction of propagation of T wave.

RESULTS

The proposed procedure provides visualization of preset frequency band with highlighting of existing resonances. The visualization can be accelerated by decrease in the number of grid nodes, thus increasing relative error of calculations. In order to find the determinant zero on frequency axis it is required to specify it by increase in the applied grid nodes.

Aiming at determination of efficiency of the proposed method of calculation of electrodynamic properties of oscillation types, it was applied as the basis for test software for calculation of higher order modes in cylindrical resonator. Relative error of calculation of resonance frequencies did not exceed $0.5 \cdot 10^{-4}$. Conversion to complex-shape resonators increases total operation time of the software.

Therefore, in the case of curvilinear orthogonal coordinates in the considered space an efficient method was obtained for determination of electrodynamic properties of higher order modes. Such coordinate system can be generated using the following algorithm. Rectangular grid is applied onto longitudinal cross section of axially symmetric space, and it is used for solution of the Laplace equation by finite difference method, provided that one portion of cross section contour is one electrode and the other portion is another electrode.

Nearly in all cases such conventional splitting of the contour is possible. The obtained potential distribution is used for determination of equipotentials (resonator boundaries are also equipotentials). Then, using properties of electrostatic field and geometrical methods, it is possible to generate the system of field lines perpendicular to equipotentials. Their crossing points are the nodes of curvilinear orthogonal coordinate system. With known positions of the nodes in reference coordinate system, for instance, cylindrical one, it is possible to calculate elements of fundamental tensor of new coordinate system.

It should be mentioned that the generated curvilinear grid has clusters in vicinity of internal protrusions of resonator boundary, that is, in the regions with high gradients of electromagnetic field, and this cluster is a natural consequence of solution of the Laplace equations.

The considered method of determination of electrodynamic properties of higher order modes, as can be readily demonstrated, can be applied for spaces without axial symmetry. In such case it is necessary to generate 3D orthogonal coordinate system, which is possible in principle, though involves some difficulties upon programming.

DISCUSSION

The selected model, of course, does not cover variety of practical slowing systems but is restricted with the structures in the form of a chain of coupled cavity resonators. Selection of such model is stipulated by possibility to carry out comprehensive numerical study of slowing systems upon relative simplicity of the obtained algorithms.

Together with the model of electrodynamic coupled cavity resonator chain another model is used in practice, which represents slowing structure in the form of a chain of coupled radio engineering circuits (Grigor`ev and Yankevich, 1975; Kalyuzhnyi et al., 2013; Novozhilov et al., 2014). Such representation of slowing systems, while providing high visualization and simplicity of algorithm, is characterized with certain disadvantages (Segerlind, 1979; Marchuk and Agoshkov, 1981), which restrict application of the circuit model (for instance, difficulty of accounting for non-symmetry of cell excitation, necessity to determine parameters of cell equivalent circuit and so on).

As mentioned above, while simulating slowing systems in the form of a chain of coupled resonators the electromagnetic fields in resonators are expanded into series in systems of eigen vector orthonormalized functions, thus providing multi-mode model. Such approach makes it possible to subdivide the problem solution into two stages: solution of boundary electrodynamics problem in order to determine eigen vector functions and determination of unknown expansion coefficients of electromagnetic field. However, most of the applied in practice slowing systems have complex-shape resonators, which complicates obtaining of analytical solution of boundary problem, and determination of eigen vector functions of such resonators requires for application of numerical methods.

CONCLUSIONS

Most numerical solutions of wave equation are based on its reduction to a set of linear algebraic equations by means of discretization (Vol'man abd Pampu, 1975). Generally four main solutions of boundary equation in cavity resonators are applied: finite difference method, variation method, integral method, and finite elements method, widely applied nowadays (Daikovskii et al., 1980). Since discretization can be considered as projection of infinite functional space, then all aforementioned methods are variants of projection-grid method (Karliner et al., 1979). Problems of improvement of accelerator tools, intention to apply more and more complex processes in accelerating systems inevitably lead to necessity of thorough theoretical investigation into all elements of designed electrodynamic systems. Numerical simulation becomes more and more powerful and multi-purpose tool of such theoretical investigations, which sometimes is significantly less expensive than natural experiment, it does not require for high expenses, makes it possible to acquire required statistics in short times and in combination with natural experiment promotes significantly development of new facilities and tools.

The tendency towards increase in currents of charged particles and total unit capacity has lead to qualitatively new requirements to numeric simulation of electrodynamic processes. One of the most significant requirements is multi-mode type of models, that is, possibility to investigate into interaction between charged particle beams and a set of electromagnetic oscillations, which are excited in operation spaces, and to study dispersion properties of structures in higher order bandwidths.

The most promising in this regard is the electrodynamic model which enables obtaining of sufficiently complete pattern of electrodynamic processes in accelerating systems. The electrodynamic model is based on expanding of required electromagnetic fields in eigen vector functions of resonators (Grigor'ev and Yankevich, 1984). In this case the whole problem is split into two relatively autonomous problems: generation of systems of eigen vector functions of resonators and determination of unknown coefficients of the expansion.

We aim at further investigations into selection of optimum accelerating structures of upon modernization of existing facilities and development of innovative charged particle accelerators with minimization of influence of higher order waves on properties of celebrated beams of charged particles. This problem is urgent for such accelerators as powerful sources of synchrotron radiation based on linear accelerators of electrons with high average capacity of accelerated beam for the purposes of industry, medicine, and environmental protection.

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