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A Numerical Method For Approximating The Time Constant Of A Measuring Circuit When Differentiating The Newton Interpolation Polynomial.

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ABSTRACT

Determination of the time constant of the measuring circuit is the main metrological task when measuring the electric capacitance at a constant current. It has been established that the most promising direction for increasing the accuracy of processing systems for measuring signals of capacitive sensors is the use of instantaneous stress values during the developing transition process. We have proposed two new algorithms for estimating the time constant, based on the approximation of derivatives using finite differences in numerical differentiation algorithms. The proposed algorithms allowed to reduce the measurement error of the electrical capacity to 0.055%.
Keywords: time constant, numerical methods, accuracy, field of efficiency.

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INTRODUCTION

Investigations of methods for measuring the parameters of electrical circuits based on the instantaneous values of transient processes have shown that the main informative parameter when processing primary information of capacitive transducers is a time constant, which can be determined by known methods of dynamic measurements [1, 2, 3].

1. By logarithm of the exponential, at a certain time:

$$\tau = -\frac{t}{\ln(\frac{(E_0 - u_{Cx}(t))}{E_0})}. \quad (1)$$

2. By using the value of the voltage function on the capacitor and its rate of change at a certain time, namely:

$$\tau = \frac{E_0 - u_{Cx}(t)}{u'_{Cx}(t)}. \quad (2)$$

3. Using the ratio of the first and second derivatives of the voltage function on the capacitance C_x at a certain time.

Really,

$$\tau = -\frac{u'_{Cx}(t)}{u''_{Cx}(t)}. \quad (3)$$

It is shown in [1, 2] that it is expedient to carry out three instantaneous voltage values during the developing transient process equidistant from each other.

In work [2], the first method of determining the time constant (1) was investigated, on the basis of which a three-point algorithm based on the logarithm of the exponent was proposed. At the same time, special attention was paid to the speed and accuracy of the measurement, and the questions of determining the area of efficiency (stability) and recommendations for the practical implementation of the algorithm under investigation remained open.

MATERIALS AND METHODS

In this paper, we consider it important to conduct research on the second (2) and third (3) methods for determining the time constant and, on their basis, the development of new computational algorithms for the signal processing system for capacitive transducers aimed at predicting the value of the controlled variable and making decisions on a real-time scale [4, 5, 6].

Methods of numerical differentiation are widely used in signal processing systems.

The 2 nd and 3 rd methods of determining the time constant (2) and (3) can be realized by approximating the derivatives using finite differences in numerical differentiation algorithms.

$\tau = \frac{E_0 - u_{Cx}(t)}{u'_{Cx}(t)}$ - 2nd method for determining the time constant;

$\tau = -\frac{u'_{Cx}(t)}{u''_{Cx}(t)}$ - third method for determining the time constant.

In [1, 2], in order to estimate the derivatives of a tabular given function $y = f(t)$ it is proposed to replace its values: $y_0, y_1, y_2 \dots y_k$ (for the values of the argument: $t_0, t_0 + \Delta t, t_0 + 2\Delta t \dots k\Delta t$) polynomial interpolation k-th degree, takes the same values as the $f(t)$ for the same values of the argument. To determine the derivative of the i-th order, it is necessary to differentiate the interpolation polynomial i times.

In this case, the finite differences of the first, second and k-th orders are necessary for the construction of interpolation polynomials defined as follows:

$$\Delta y_i = y_{i+1} - y_i; \quad (4)$$

$$\Delta^2 y_i = y_{i+1} - \Delta y_i; \quad (5)$$
$$\Delta^k y_i = \Delta^{k-1} y_{i+1} - \Delta^{k-1} y_i. \quad (6)$$

It is convenient to represent finite differences of the 1st and 2nd order in the form of Table 1 (for k = 2).

To estimate the first derivative we used interpolation polynomials of Newton, Stirling, and Bessel.

After differentiating the interpolation polynomials of Newton, Stirling, and Bessel with respect to the variable t for the values of the argument t = t_0, the most common numerical differentiation algorithms of the first and second orders are presented in Table 2.

RESULTS AND DISCUSSION

The analysis of the relations of the Newton, Stirling and Bessel interpolation polynomials obtained by differentiation (Table 2) showed that the algorithms for numerical differentiation based on the Stirling and Bessel polynomials have a significant drawback - the inability to use the derivative estimate in real time.

Table 1: Finite differences of the 1st and 2nd orders

t	y	Δy	$\Delta^2 y$
t_0	y_0		
		Δy_0	
$t_0+\Delta t$	y_1		$\Delta^2 y_0$
		Δy_1	
$t_0+2\Delta t$	y_2		

Table 2: Estimation of the first derivative in the differentiation of interpolation polynomials of Newton, Stirling and Bessel

Polynomial	1st order	2nd order
Newton	$\tilde{y}'(t_0) = \frac{y_1 - y_{-1}}{2\Delta t}$	$\tilde{y}'(t_0) = \frac{3y_0 - 4y_1 + y_{-2}}{2\Delta t}$
Stirling	$\tilde{y}'(t_0) = \frac{y_1 - y_{-1}}{\Delta t}$	$\tilde{y}'(t_0) = \frac{-y_2 + 8y_1 - 8y_{-1} + y_{-2}}{4\Delta t}$
Bessel	$\tilde{y}'(t_0) = \frac{y_1 - y_0}{\Delta t}$	$\tilde{y}'(t_0) = \frac{-y_2 + 5y_1 - 3y_0 + y_{-1}}{4\Delta t}$

This is due to the fact that to evaluate the derivative at a given, specific moment in time, the signal values are used at times ahead of this time. Therefore, the use of numerical differentiation algorithms based on Stirling and Bessel polynomials in relation to the tasks of monitoring, processing and analyzing measuring signals that are solved in real time is not possible.

For our research, in particular, the possibility of predicting the measured value, and on its basis of decision-making on a real-time scale, algorithms for numerical differentiation of the derivative estimate based on the Newton interpolation polynomial are suitable, since only the previous samples are used in them. An estimate of the derivative at a point y_0 s performed from the values of the observed function $y_0, y_{-1}, y_{-2} \dots y_{-k}$ at moments of time t_0 and at previous times $t_0 - \Delta t, t_0 - 2\Delta t, \dots t_0 - k\Delta t$.

The test method can be used to implement new methods for measuring the level from the value of the capacitance sensor capacitance.

The Newton interpolation polynomial with finite differences for forward interpolation is as follows:

$$P_n(x) = P_n(x_0 + ht) = y_0 + \frac{\Delta y_0}{1!}t + \frac{\Delta^2 y_0}{2!}t(t-1) + \frac{\Delta^3 y_0}{3!}t(t-1)(t-2) + \cdots + \frac{\Delta^n y_0}{n!}t(t-1)\ldots(t-n+1)$$
(7)

In this case, a one-way forum for calculating the first derivative f' during differentiation (7) and having an n-th order accuracy:

$$f'(x_0) \approx \frac{1}{h}\sum_{j=1}^{n}\frac{(-1)^{j-1}}{j}\Delta^j y_0 \quad (8)$$

For n = 2, we obtain from (8) that:

$$f'(x_0) \approx \frac{1}{2h}(-3f(x_0) + 4f(x_1) - f(x_2)) \quad (9)$$

Consider the derivative of Newton's interpolation polynomial:

$$\tilde{y}'(t_0) = \frac{1}{\Delta t}(\Delta y_{-1} + \frac{1}{2}\Delta^2 y_{-2} + \cdots). \quad (10)$$

Consider the first term of expression (7):

$$\tilde{y}'(t_0) = \frac{1}{\Delta t}(\Delta y_{-1}) = \frac{y_0 - y_{-1}}{\Delta t}. \quad (11)$$

The second term (7) can be written:

$$\Delta^2 y_{-2} = y_0 - 2y_{-1} + y_{-2}. \quad (12)$$

We substitute the expression (12) into the estimate of the first derivative (7), we obtain:

$$\tilde{y}'(t_0) = \frac{1}{\Delta t}\left(y_0 - y_{-1} + \frac{y_0 - 2y_{-1} + y_{-2}}{2}\right) = \frac{2y_0 - 2y_{-1} + y_0 - 2y_{-1} + y_{-2}}{2\Delta t} \quad (13)$$

or

$$\tilde{y}'(t_0) = \frac{-y_3 - 4y_2 - 3y_1}{2\Delta t}, \quad (14)$$

where y_1, y_2, y_3 – the value of a differentiable function at equidistant times t_1, t_2, t_3 respectively;
Δt – step of differentiation, which is:

$$\Delta t = t_2 - t_1 = t_3 - t_2. \quad (15)$$

Using the estimate of the first derivative (14) as applied to the second method of determining the time constant of expression (2), a discrete measurement measurement algorithm $\tilde{\tau}$ on three points:

$$\tilde{\tau} = \frac{2\Delta t \cdot (E_0 - u(t_1))}{-u(t_3) + 4u(t_2) - 3u(t_1)}, \quad (16)$$

where E_0 – Emf voltage source;
$\tilde{\tau}$ - Estimation of time constant IC;
$u(t_1), u(t_2), u(t_3)$ - values of voltage function counts at time point t_1, t_2, t_3 respectively during the transient process.

CONCLUSION

Thus, a three-point algorithm for estimating the time constant based on the measurement of the instantaneous values of the transient process is proposed with the possibility of predicting a controlled variable and making decisions in real time that can be used for high-precision measurement of the electrical capacitance of primary level converters of various liquids.

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